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ABSTRACT: This note points out the connections between a problem raised in Miltenburg(1989) and the asssignation of seats in electoral processes. The coincidence between the algorithm proposed by Miltenburg for the problem and the Method of Largest Fractions (LF) allows us to exploit the known properties of LF to generalize the procedure of Miltenburg. The note also points out the possibility of using other electoral procedures to solve the problem.

Keywords: PRODUCTION/SCHEDULING_LINE BALANCING; INVENTORY/PRODUCTION_JUST-IN-TIME; APPORTIONMENT PROBLEM

Miltenburg (1989) proposes a framework for scheduling just-in-time production systems which includes a model for determining the sequence of production of different articles on a line and proposes different algorithms to solve it.

An important part of Miltenburg's work consists in finding, given a vector $\boldsymbol{R}$ (such that $\Sigma_i r_i = 1$) and a positive integer $\boldsymbol{k}$, an integer vector $\boldsymbol{X(k)}$, such that $\Sigma_i x_{ik}$ = k and which is at minimum quadratic distance from the vector $\boldsymbol{kR}$. Miltenburg explains and justifies at length the algorithm wich makes it possible to obtain $\boldsymbol{X(k)}$, given k and R.

This problem is a particular case of the apportionment problem, wich consists in determining an integer vector, with the sum of its components equal to $\boldsymbol{k}$, as near as possible to a vector of generally non- integers components (quotas) wich result from sharing out k proportionally between different options. This problem arises in many real circumstances, and in particular in political processes, such as the distribution of the total number of seats of a house of representatives among different constituencies or the assignment of seats to the political parties contesting an election, when it is desired that the distribution should be as proportional as possible (e.g. the number of seats proportional to the population of the constituency or to the number of votes obtained by the party, according to the case).

This problem has been widely studied; see, for example, Lucas (1978). Obviously, the different functions of discrepancy between pairs of vectors lead to different methods (though methods which minimize more than one function of discrepancy are not rare); in practice a function of discrepancy is not normally used as a premise, but a method is chosen bearing in mind its different properties.

One of the oldest methods for assigning seats (going back to at least 1792, when it was proposed by Alexander Hamilton) is the Method of Largest Fractions (LF), also known as the Method of Greatest Remainders, the Hare Quota Method, the Method of Computed Ratio and Hamilton's or Vinton's Method. It consists in assigning to each option the integer part of the quota and the rest of the seats, successively, according to the decreasing order of the fractional part of the quotas. As can be seen, this is exactly the procedure used by Miltenburg in his article. LF has the property of minimizing the following functions:

$$\sum_i (x_{ik} - kr_i)^2$$

$$\sum_i |x_{ik} - kr_i|$$

$$\sum_i \|X(k) - kR\|$$

where $\|Y\|$ expresses the norm l_p of the vector Y

$$max_i |x_{ik} - kr_i|$$

by which, as can be seen, Miltenburg's procedure, which the author proposes for the quadratic distance, may be extended to other functions of discrepancy.

Unfortunately, LF, as occurs with any other known method of assignation of seats, does not have all the properties which would seem desirable. In particular, LF is not house monotone, i. e. for some given r_i it is not always true that $x_{ik} \leq x_{i,k+1}$ and therefore it may occur that $x_{ik} > x_{i,k+1}$, which means that as the number of seats to be distributed increases, the number of seats attributed to an option *decreases*. This affected the states of Alabama, Colorado and Maine in a certain distribution of seats, and this fact came to be known as the Alabama paradox.

The Alabama paradox prevents the Miltenburg procedure from providing the optimum in all cases and makes it necessary to resort to heuristic procedures.

The connections between the problem of sequences dealt with by Miltenburg and the apportionmemt problem do not stop here. As well as LF, there are of course other procedures for assigning seats, some of which minimize certain functions of discrepancy.

For example, the Major Fractions Method minimizes the following functions:

$$\sum_i kr_i\left(\frac{x_{ik}}{kr_i}-1\right)^2$$

$$\sum_i \frac{(x_{ik}-kr_i)^2}{kr_i}$$

$$\sum_i \frac{x_{ik}(x_{ik}-kr_i)^2}{(x_{ik}kr_i)^{\frac{1}{2}}}$$

and the Equal Proportions Method, among others:

$$\sum_i \frac{(x_{ik}-kr_i)^2}{x_{ik}}$$

Indeed, the use of the results established for the apportionment problem may be useful for solving the problem considered by Miltenburg.

References.

MILTENBURG, J., "Level Schedules for Mixed-model Assembly Lines in Just-In-Time Production Systems"., *Management Science*, 35, 2(1989), pp. 192-207.

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