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A note on one-processor scheduling with equal task lengths and with nonsymmetric, nonlinear earliness and tardiness penalties

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A NOTE ON ONE-PROCESSOR SCHEDULING WITH EQUAL TASK LENGTHS AND WITH NONSYMMETRIC, NONLINEAR EARLINESS AND TARDINESS PENALTIES

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ABSTRACT

We consider the problem of determining an optimal schedule for a number of tasks of equal length to be processed nonpreemptively on one single machine; the objective is to minimize the sum for all the tasks of a convex function of the discrepancy between their completion times and the respective due dates. It is shown that an optimal solution always exists in which the tasks are processed in order of EDDs (earliest due dates), and some solution procedures are proposed, for one of which we present a synthesis of the computational experience obtained.

1. Introduction

The problem dealt with in this note is the following: there are n tasks of equal length (thereby allowing us, without forgoing generality, to consider task length as being equal to one time unit) to be processed nonpreemptively on a single processor which can only carry out one task at a time; a schedule is an assignment to each task of a completion time, c_i , such that no two tasks overlap in their execution; each task has an associated due date d_i and, given a schedule, a corresponding earliness, equal to $\max(0,d_i-c_i)$, and tardiness, equal to $\max(0,c_i-d_i)$. As a notational convenience whe shall assume that $d_i \le d_{i+1}$; whe shall also assume that all the tasks are available at time 0 (or that their release dates, r_i , fullfil the condition $r_i \le i-1$). Given an earliness penalty function g and a tardiness penalty function h, both nonnegative, convex and such that $g(0)=h(0)=0$, we wish to determine a schedule which minimizes the total penalty obtained by summing all the individual earliness and tardiness penalties.

Without loss of generality, we shall consider a convex, nonnegative function f (of the difference $\delta_i=c_i-d_i$) such that $f(0)=0$. As will be seen subsequently, the only essential feature for optimization is that it be convex, although any appropriate function for the sequencing under consideration here must clearly have the other two properties. In short, our task is to minimize the function:

$$z=\sum_{i=1}^{n} f(\delta_i)=\sum_{i=1}^{n} f(c_i-d_i)$$

This problem is a particular case of that of scheduling with a single processor, on the subject of which several works have been published. In Garey, Tarjan and Wilfong (1988) it is dealt with in full with symmetric and linear earliness and tardiness penalties; the authors observe that researchers have come to consider scheduling problems in which both tardiness and earliness are penalized, in contrast with the previous approach of penalizing only tardiness (which amounted, in fact, to judging it no more desirable to finish a task on its due date than at any other previous time). The treatment of earliness alongside tardiness as a magnitude with an associated penalty is in keeping with just-in-time policies, which probably accounts for its acceptance within models. Nevertheless, equal importance for earliness and tardiness is seldom justified, just as their contribution to the objective function will rarely be linear. Positive earliness generates storage costs (which can be considered proportional to earliness), but the repercussions of positive tardiness are entirely different, and not necessarily proportional to its value. Thus, in practice the appropriate function f for modelling the objective will tend to be neither symmetric nor linear.

In the paper mentioned above, Garey, Tarjan and Wilfong consider the symmetric case in which task lengths are equal. This case is of particular interest, since it has many features in common with the problem of sequencing units of variants of a product on an assembly line with the aim of regularizing the appearance of variants of the product. This problem, which Kubiak (1993) named the PRV (Product Rate Variation) problem, was introduced into the literature by Miltenburg (1989) as a development of Monden's precursory work (1983), and has been researched by several authors. The original approach is to measure regularity in terms of the discrepancies between real and ideal production totals, and the objective is to minimize a given function of these discrepancies, such as the sum of their squares. As a route towards the optimum of this function (in other words, a heuristic procedure), Inman and Bulfin (1991) propose seeking an optimal solution to the problem of minimizing the sum of the squares of the differences between the units' completion times and ideal dates determined to that effect. Inman and Bulfin then note that this optimization problem coincides with the scheduling problem with a single processor and equal task lengths; they solve it by using the EDD rule, that is, by putting the units in order of first to last due dates. In fact, for the case of equal task lengths and the objective of minimizing the sum of the absolute δ_i values, it is shown in Garey, Tarjan and Wilfong (1988) that an optimal solution responding to the EDD rule always exists. In Inman and Bulfin (1991) we see that the same interchange argument used in Garey, Tarjan and Wilfong (1988) is valid for the sum-of-the-squares function.

In this note we shall demonstrate (in point 2) that this argument is also valid for any function f with the properties specified above. In point 3 we discuss the procedures for determining an optimum schedule with a set EDD order, and present a computational experience. Finally, point 4 offers some comments on the minimax problem and some very brief conclusions.

2. EDD minimizes $\sum_{i=1}^{n} f(\delta_i)$

Let a convex function f and four values (p,q,r,s) be such that

$$p<q<s$$
$$p<r<s$$

with $q=p+\varepsilon$ and $r=s-\varepsilon$, where, of course, $0\le\varepsilon\le s-p$. We first show that the following proposition is true:

Proposition: $f(q)+f(r) \leq f(p)+f(s)$

Indeed,

$$q = \frac{s-p-\varepsilon}{s-p} p + \frac{\varepsilon}{s-p} s$$
$$r = \frac{\varepsilon}{s-p} p + \frac{s-p-\varepsilon}{s-p} s$$

So, for the convexity of f :

$$f(q) \leq \frac{s-p-\varepsilon}{s-p} f(p) + \frac{\varepsilon}{s-p} f(s)$$
$$f(r) \leq \frac{\varepsilon}{s-p} f(p) + \frac{s-p-\varepsilon}{s-p} f(s)$$

and, therefore:

$$f(q)+f(r) \leq f(p)+f(s)$$

We shall demonstrate that if we have a solution with $d_i \leq d_j$ and $c_i > c_j$, the solution obtained by permutation of the two tasks is not worse than the initial solution (permutation is always possible, since the two tasks are of equal length; moreover, this permutation does not alter the completion times of the other tasks).

In fact, we can write:

$$d_j = d_i + \varepsilon$$
$$c_i = c_j + \eta$$

with $\varepsilon > 0$ and $\eta > 0$ and, therefore, the inequality

$$f(c_i - d_j) + f(c_j - d_i) \leq f(c_j - d_j) + f(c_i - d_i)$$

is equivalent to:

$$f(c_i-d_i-\varepsilon)+f(c_i-d_i-\eta)\leq f(c_i-d_i-\eta-\varepsilon)+f(c_i-d_i)$$

which is fulfilled by virtue of the above proposition.

3. Obtaining an optimal schedule

Once it has been shown that an optimal solution always exists in which the tasks are arranged in accordance with the EDD rule, it is relatively simple to determine an optimal schedule.

On the one hand, there is the possibility of adapting the algorithm proposed in Garey, Tarjan and Wilfong (1988) for reaching an optimal solution when the order of the tasks is set. The authors present a detailed description of their algorithm for linear and symmetric penalties, and also state several generalizations. In the problem with which we are concerned, the adapted algorithm would have to include a procedure for the optimization of nonlinear functions of a variable.

Alternatively, the problem can be modelled by means of mathematical programming:

$$\begin{aligned}
&[MIN]z=\sum_{i=1}^{n} f(\delta_i)\\
&c_i+1\leq c_{i+1} \quad 1\leq i\leq n-1\\
&c_i=d_i+\delta_i \quad 1\leq i\leq n\\
&c_1\geq 1
\end{aligned}$$

We are dealing, then, with a nonlinear program with a convex objective function and linear restrictions which can be solved with any standard algorithm (of course, we can use the starting times instead of the completion times).

In practice, we will frequently use different expressions to define the penalties associated with earliness and tardiness, which makes it convenient to use the following formulation:

$$[MIN]z=\sum_{i=1}^{n}[f^{-}(\delta_i^{-})+f^{+}(\delta_i^{+})]$$

$$c_i+1\leq c_{i+1} \qquad 1\leq i\leq n-1$$

$$c_i=d_i+\delta_i^{+}-\delta_i^{-} \qquad 1\leq i\leq n$$

$$c_1\geq 1;\ \delta_i^{+},\delta_i^{-}\geq 0\,\forall i$$

With this model, we performed a computational experience consisting of three experiments.

Experiment 1

The penalties associated with earliness and tardiness were defined as:

$$f^{-}(\delta^{-})=\alpha^{-}\delta^{-}\,;f^{+}(\delta^{+})=\alpha^{+}+\beta(\delta^{+})^{p}$$

For the parameters of these functions, the eight sets of values shown in **table 1** were used.

	S1	S2	S3	S4	S5	S6	S7	S8
α^{-}	0	1	2	1	1	1	1	1
α^{+}	1	2	1	1	1	1	2	2
β	0	0	0	0	1	1	1	1
p	0	0	0	0	2	4	2	4

Table 1

The instances proposed give five values for the number of tasks to be programmed (n): 20, 40, 60, 80 and 100; and the d_i values were obtained by means of a succession of numbers $(x_j,\ 1\leq j\leq n)$ generated by simulation as and 2), by performing:

$$d_i=\sum_{j=1}^{i} x_j$$

This yielded a total of 120 instances (grouped in 15 blocks, defined in terms of n and λ, each of them consisting of eight instances associated with the sets of values for the parameters), modelled in the language GAMS and solved with the optimizers BDMLP (linear functions) and MINOS (nonlinear functions) on a 66 Mz PC-486.

The total times required for solving each block of problems are shown in **table 2**.

	n=20	n=40	n=60	n=80	n=100
λ=0.5	50 s	54 s	58 s	62 s	65 s
λ=1.0	50 s	54 s	58 s	61 s	65 s
λ=2.0	50 s	53 s	56 s	59 s	61 s

Table 2

These times include file management, compilation of the model and the solution of the problems.

The time employed on each instance (excluding file management and compilation) varied between 2.9 and 6.06 s respectively for an instance with n=20 and a linear function and a for an instance with n=100 and a nonlinear function.

The data and the solution for one of the instances are shown in **figure 1**.

Task number	Due date	Starting time	Task number	Due date	Starting time
1	1.60	0	11	23.61	22.61
2	1.62	1	12	26.28	24.28
3	5.32	3.91	13	26.85	25.28
4	5.91	4.91	14	27.28	26.28
5	7.84	6.84	15	29.96	28.96
6	10.84	9.84	16	31.40	30.01
7	11.84	10.84	17	32.01	31.01
8	15.39	14.39	18	35.86	34.83
9	22.68	20.61	19	36.83	35.83
10	23.23	21.61	20	38.03	37.03

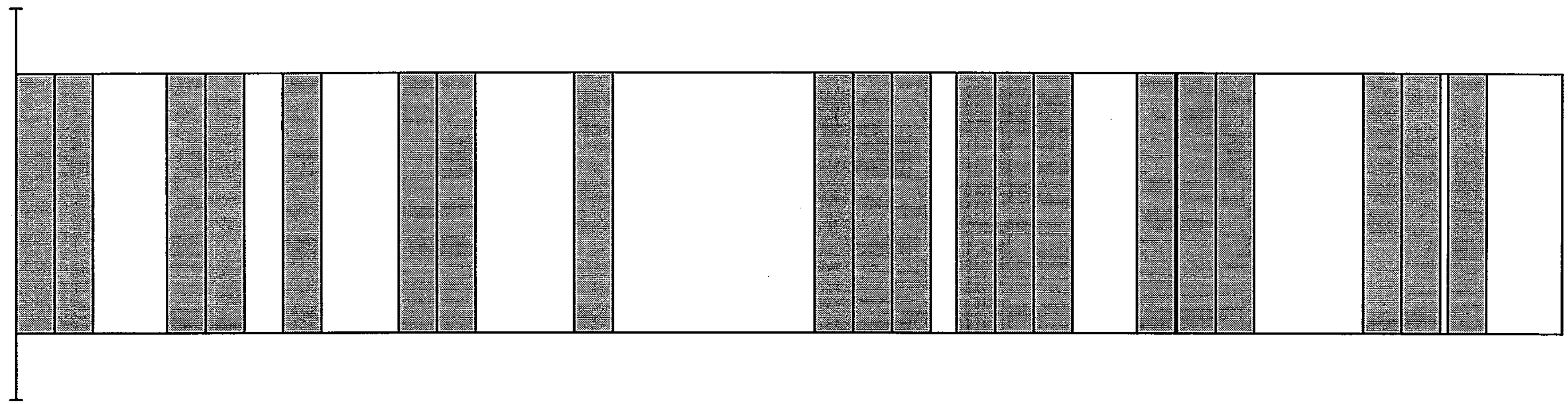

Figure 1

Data and solution for one of the instances of experiment 1

Task number	Due date	Starting time	Task number	Due date	Starting time
1	1.60	0	11	23.61	22.61
2	1.62	1	12	26.28	24.28
3	5.32	3.91	13	26.85	25.28
4	5.91	4.91	14	27.28	26.28
5	7.84	6.84	15	29.96	28.96
6	10.84	9.84	16	31.40	30.01
7	11.84	10.84	17	32.01	31.01
8	15.39	14.39	18	35.86	34.83
9	22.68	20.61	19	36.83	35.83
10	23.23	21.61	20	38.03	37.03

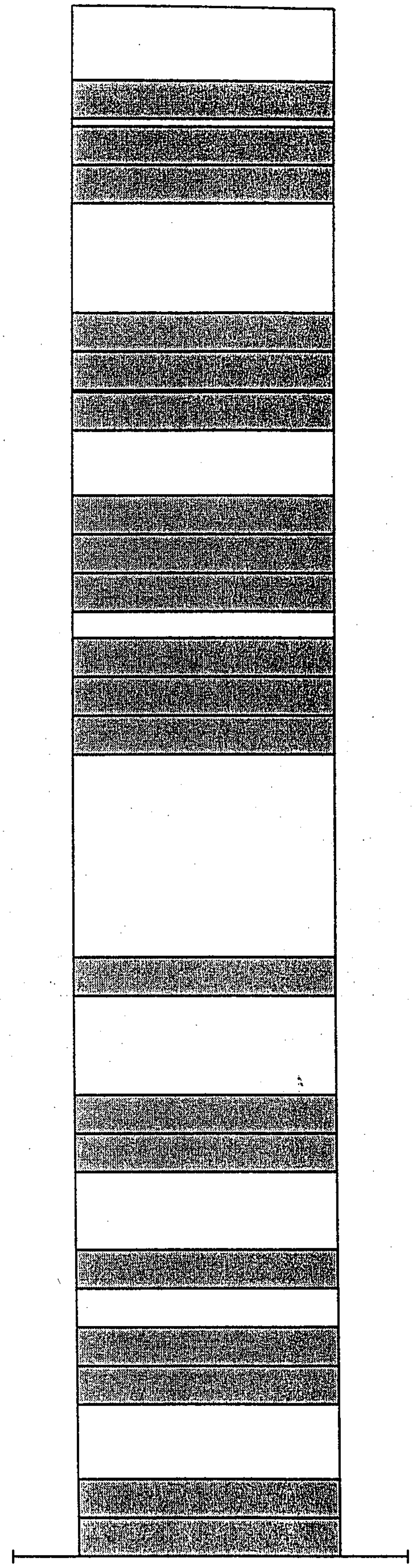

Figure 1

Data and solution for one of the instances of experiment 1

Experiment 2

The same penalty function was used, but with only the first four sets of values for the parameters (the linear functions).

The values set for n were 50, 100, 150, 200 and 250, and the d_i values were obtained in the same way as in the previous experiment.

This therefore gave a total of 60 instances, grouped as before in 15 blocks, the solution of which required the times that are displayed in **table 3**.

	n=50	n=100	n=150	n=200	n=250
λ=0.5	28 s	32 s	39 s	46 s	55 s
λ=1.0	28 s	32 s	34 s	38 s	48 s
λ=2.0	28 s	30 s	32 s	36 s	39 s

Table 3

The time for each model varied between 3.62 and 7.03 s for instances with n=20 and n=250 respectively.

Experiment 3

In this experiment we set a common due date for all the tasks $(d_i = d_0\ \forall i)$. It consisted of a total of 160 instances grouped in 5 blocks. Each block, defined in terms of $n\,(n=20,40,60,80,100)$, contains 32 instances, formed by combining the eight sets of values for the parameters and four values for d_0 .

Table 4 features the four values for d_0 adopted for each value of n and **table 5** contains the overall times required to solve each block.

n=20	10	20	50	100
n=20	20	40	100	200
n=20	30	60	150	300
n=20	50	80	200	400
n=20	50	100	250	500

Table 4

n=20	n=20	n=20	n=20	n=20
179 s	193 s	211 s	227 s	242 s

Table 5

The time for each model varied between 3.13 and 6.58 s respectively for instances with n=20 and a linear function and n=100 and a nonlinear function.

4. Remarks and conclusions

If we regard the problem hitherto discussed as being of the *minisum* type, when the objective function we are seeking to minimize takes the form $\max_i f(\delta_i)$, we can state that the problem is of the *minimax* type. *Minimax* problems with an equal task length are -- see Garey, Tarjan and Wilfong (1988) -- a particular case of the problem studied in Sidney (1977) and Lakshminarayan et al. (1978), for which an optimal solution always exists that follows the EDD rule (in fact, this is easily seen directly, by applying the same exchange argument as for the *minisum* case, except that in the *minimax* case it is sufficient for the function f to be quasi-convex).

For *minisum* problems with equal lengths for all tasks, this note generalizes, for any convex function of the discrepancies between completion times and due dates, the results obtained by Garey, Tarjan and Wilfong (1988), according to which (for the absolute-value function of the discrepancies) there is always an optimal solution which follows the EDD rule. This enables us to solve the problem of finding an optimal schedule through standard mathematical programming, and the computational experience obtained suggests that this approach is practical for problems on an industrial scale.

The proposed approach also enables us to solve the case in which we have for the completion time of each task a window $[a_i,b_i]$ $(a_i \leq b_i)$ whitin which the task can shift whitout penalty -- the earliness and the tardiness are, then, respectively equal to $\max(0,a_i-c_i)$ and $\max(0,c_i-b_i)$ --; of course, we also can use the model for finding an optimum schedule when the tasks must be executed in a fixed order.

References

BROOKE, A., KENDRICK, D. AND MEERAUS, A., *GAMS. A user's guide*, The Scientific Press, South San Francisco, CA, 1988.

GAREY, M.R., TARJAN, R.E. AND WILFONG, G.T., "One-processor scheduling with symmetric earliness and tardiness penalties", *Mathematics of Operations Research*, 13 (1988) 2, 330-348.

INMAN, R.R. AND BULFIN, R.L., "Sequencing JIT mixed-model assembly lines", *Management Science*, 35 (1991) 7, 901-904.

KUBIAK, W., "Minimizing variations of productions rates in just-in-time systems: A survey", *European Journal of Operational Research*, 66 (1993), 259-271.

LAKSHMINARAYAN, S., LAKSKMANAN, R., PAPINEAU, R. AND ROCHETTE, R.,"Optimal single-machine scheduling with earliness and tardiness penalties", *Operations Research*, 26 (1978), 1079-1082.

MILTENBURG, J.G., "Level schedules for mixed-model assembly lines in just-in-time production systems", *Management Science*, 35 (1989) 2, 192-207.

MONDEN, Y., *Toyota Production System*, Institute of Industrial Engineers Press, Norcross, GA, 1983.

SIDNEY, J. B., "Optimal single-machine scheduling with earliness and tardiness penalties", *Operations Research*, 25 (1977), 62-69.