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A comment on 'A simple sequencing algorithm for mixed-model assembly lines in Just-In-Time production systems'

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TÍTOL:

"A COMMENT ON 'A SIMPLE SEQUENCING ALGORITHM FOR MIXED-MODEL ASSEMBLY LINES IN JUST-IN-TIME PRODUCTION SYSTEMS"

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A comment on "A comment on 'A simple sequencing algorithm for mixed-model assembly lines in just-in time production systems'"

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Abstract

A note was recently published by Yeomans (1994) comparing two heuristics, DC (1993) and H2 (1989), for the level schedule problem in mixed-model just-in-time assembly systems. In it, he comes to the conclusion that DC is an implementation of H2 with O(nD) complexity. The present note shows that heuristics H2 and DC do not produce the same solution in all cases and that they are therefore two distinct heuristics. The results of a brief computational experiment are included.

Key words: Assembly systems; Just-in-time

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A comment on "A comment on 'A simple sequencing algorithm for mixed-model assembly lines in just-in time production systems'"

Yeomans [9] recently published some comments on a heuristic, DC, proposed by Ding and Cheng [2, 3] for the level schedule problem in mixed-model just-in time (JIT) assembly systems. The heuristic DC is an O(nD) procedure where n is the number of different products and D is the total demand for all of the products to be produced. One of Yeomans's main criticisms concerns the comparison Ding and Cheng make between the heuristics DC and M-A3H2, the latter being described in [7]. The solutions provided for the two procedures are of similar quality but the computational burden is significantly lower in DC heuristic. Yeomans mentions the existence in the literature of procedures with higher-quality solutions than M-A3H2, in particular those providing an optimal solution (implicit enumeration and dynamic programing algorithms [8], reduction to and assignment problem [5, 6], etc.); therefore optimal level production sequences could be determined on $O(D^3)$ time. As for heuristics with similar quality solutions to DC, Yeomans cites the heuristics H1 and H2, proposed by Miltenburg [7], with O(nD) and $O(n^2D)$ complexities respectively, and that proposed by Inman and Bulfin [4], with O(nD) complexity.

Yeomans asserts that H2 can solve large (n=20, D=5000) problems in less than three CPU seconds, and that it is a very easy heuristic for practitioners to understand and implement, the key attribute for heuristics as described by Ding and Cheng. Moreover, given that DC is a two-stage procedure which will produce identical sequences to the two-stage heuristic H2, DC is really an O(nD) implementation of H2. This proves unimportant, since H2 can efficiently solve all problems of realistic size using only negligible processing time. Yeomans concludes that the complexity reduction of DC heuristic provides a mainly theoretical benefit.

Unfortunately, elementary computational experiments plainly show that H2 and DC do not produce identical sequences in all cases. For example, in the instance n=3, D=25, d_1=14, d_2=10, d_3=1 the sequences obtained are:

H2 1-2-1-2-1-2-1-1-2-1-2-1-3-1-2-1-2-1-1-2-1-2-1-2-1

DC 1-2-1-2-1-2-1-1-2-1-2-1-1-2-3-1-2-1-1-2-1-2-1-2-1

the values of the objective function:

$$\sum_{k=1}^{D} \sum_{i=1}^{n} (x_{i,k} - k \cdot r_i)^2 = \sum_{k=1}^{D} V_k(x_{1,k}, x_{2,k}, \ldots, x_{n,k}) = \sum_{k=1}^{D} V_k(\sigma_k)$$

being 7.04 (H2) and 7.44 (DC), respectively.

The basic reason for this is that DC is not strictly a two-stage procedure, despite statements to that effect in [2] and [3]. Using the notation in [2], the heuristic DC selects, for each value of k, first a product s and then, depending on that choice, a second product t; finally it analyses which order (st or ts) is better and on the basis of the result decides to sequence product s or product t in position k. This procedure cannot coincide in all cases with evaluating all possible st pairs, choosing the lowest value st pair and the sequencing of s in position k. The equivalence of the two procedures is apparently demonstrated in [2] and [3], but the proof falls into error (point 4 on page 34 of [2]). There it is stated that with the procedure:

$$4\cdot[x_{s,k-1} - (k+½)\cdot r_s] - 4\cdot[x_{a,k-1} - (k+½)\cdot r_a] +$$
$$+ 2\cdot[x_{t,k-1} - (k+1)\cdot r_t] - 2\cdot[x_{s,k-1} - (k+1)\cdot r_s] \leq 0$$

where s and t are the products selected and a is a product different from s or t. In the position k=13, in the exemple given, we have $x_{1,12}=7$, $x_{2,12}=5$, $x_{3,12}=0$, s=1 and t=2, and the additional DC rule gives priority to s. If we consider a=3, the only product different from s and t, the left hand side of the inequality takes the value 0.4, which is not lower than or equal to 0. The error is caused by the fact that in the choice of t:

$$x_{t,k-1} - (k+1)\cdot r_t \leq x_{s,k-1} + 1 - (k+1)\cdot r_s$$

prevails and not:

$$x_{t,k-1} - (k+1)\cdot r_t \leq x_{s,k-1} - (k+1)\cdot r_s$$

as is assumed in the proof.

It can be shown that values of the quadratic function, in positions 13 and 14, for all possible continuations of subsequence σ, of the products sequenced between positions 1 and 12 (σ=121212112121) are:

$V_{13}(\sigma 1) = 0.8288$ | $V_{14}(\sigma 11) = 2.0192$
$V_{13}(\sigma 2) = 0.9888$ | $V_{14}(\sigma 12) = 0.4992 = V_{14}(\sigma 21)$
$V_{13}(\sigma 3) = 0.3488$ | $V_{14}(\sigma 13) = 0.5792 = V_{14}(\sigma 31)$
$V_{14}(\sigma 22) = 2.9792$
$V_{14}(\sigma 23) = 1.0592 = V_{14}(\sigma 32)$

Therefore $V_{13}(\sigma 1)+V_{14}(\sigma 12)=1.328>V_{13}(\sigma 3)+V_{14}(\sigma 31)=0.928$. A two-stage procedure will sequence the product 3 in position k=13, preferably at 1 or 2. Furthermore, 1.328-0.928=0.4, the value mentioned above, which coincides with the difference between the values of the objective functions 7.44-7.04=0.4, since the sequences of H2 and DC only differ, with regard to the units sequenced from position 1, in positions 13 and 14.

We conclude, then, that DC is not an implementation of H2 but a very similar heuristic to it with O(nD) complexity. In most circumstances, but not all, DC behaves as a two-stage procedure.

H2 is a heuristic which provides high-quality solutions, but when the solutions of H2 and DC are different, that given by H2 is not always better. We performed a computational experiment with n=4 and D between 45 and 50, the instances studied being those which correspond to the values n and D indicated with different groups of values (d_1, d_2, d_3, d_4). Table 1 shows the percentage of optima attained by each heuristic. The optimal solution was determined by means of a procedure based on BDP (bounded dynamic programming [1]). Also shown is the mean unitary resolution time on a PC 486 (66 Mz). Of the 4756 instances, in 929 (19.5 %) of them H2 obtained a better solution than DC, and in 278 (5.9 %) DC gave a better solution than H2. In the remaining 3549 (74.6 %) the solutions yielded by the two procedures had objective functions with identical values. In 333 (7.0 %) instances, neither of the two procedures obtained an optimal solution.

TABLE 1

n	D	problem instances	% opt.H2	% opt.DC	elapsed time (sec.) H2	DC	BDP
4	45	672	88.5	72.2	0.368	0.051	1.785
4	46	720	89.9	79.6	0.379	0.050	1.818
4	47	764	85.2	77.9	0.388	0.053	2.076
4	48	816	89.3	70.7	0.392	0.054	2.043
4	49	864	86.1	72.8	0.407	0.054	2.210
4	50	920	87.5	73.5	0.415	0.056	2.078
GLOBAL		4756	87.7	74.3	0.393	0.053	2.015

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A comment on 'A simple sequencing algorithm for mixed-model assembly lines in just-in time production systems'

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Abstract

Ding & Cheng (1993) presented a heuristic, with complexity O(nD), for the level schedule problem in mixed-model just-in-time assembly systems which they claimed achieves the same result as H2 heuristic proposed by Miltenburg (1989), with complexity $O(n^2D)$. The present note shows that heuristics H2 and DC do not produce the same solution in all cases and that they are therefore two distinct heuristics. The results of a brief computational experiment are included.

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Miltenburg [4] has presented a sequencing algorithm based an a heuristic, H2, for the level schedule problem in mixed-model just-in time (JIT) assembly systems, that minimizes the total variation over the next two stages by enumerating all possible pairs. Ding & Cheng [2, 3] have presented a simpler, quicker heuristic, DC, which they claimed achieves the same result as H2. Yeomans [6] has pointed out alternative approaches for the problem (including optimal [5] and heuristic methods) while opining that DC is just an implementation of H2.

Unfortunately, elementary computational experiments plainly show that H2 and DC do not produce identical sequences in all cases. For example, in the instance n=3, D=25, d_1=14, d_2=10, d_3=1 the sequences obtained are:

H2 1-2-1-2-1-2-1-1-2-1-2-1-3-1-2-1-2-1-1-2-1-2-1-2-1

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the values of the objective function:

$$\sum_{k=1}^{D} \sum_{i=1}^{n} (x_{i,k} - k \cdot r_i)^2 = \sum_{k=1}^{D} V_k(x_{1,k}, x_{2,k}, \ldots, x_{n,k}) = \sum_{k=1}^{D} V_k(\sigma_k)$$

being 7.04 (H2) and 7.44 (DC), respectively.

The basic reason for this is that DC is not strictly a two-stage procedure, despite statements to that effect in [2] and [3]. Using the notation in [2], the heuristic DC selects, for each value of k, first a product s and then, depending on that choice, a second product t; finally it analyses which order (st or ts) is better and on the basis of the result decides to sequence product s or product t in position k. This procedure cannot coincide in all cases with evaluating all possible st pairs, choosing the lowest value st pair and the sequencing of s in position k. The equivalence of the two procedures is apparently demonstrated in [2] and [3], but the proof falls into error (point 4 on page 34 of [2]). There it is stated that with the procedure:

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where s and t are the products selected and a is a product different from s or t. In the position k=13, in the example given, we have $x_{1,12}=7$, $x_{2,12}=5$, $x_{3,12}=0$, s=1 and t=2, and the additional DC rule gives priority to s. If we consider a=3, the only product different from s and t, the left hand side of the inequality takes the value 0.4, which is not lower than or equal to 0. The error is caused by the fact that in the choice of t:

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LAS MISERIAS DEL LAUREL
por Victor da Lonardi

Un "referee" de un artículo que sometimos a una revista atrajo nuestra atención sobre una nota publicada firmada por Yeomans [1]. Como era de esperar dicha nota no tenía relación alguna con el artículo sometido, pero al leerla con atención nos sorprendió su inusitada virulencia. Yeomans ponía de chupa de dómine un artículo anterior de Ding y Cheng [2]. Estos últimos autores habían tenido la osadía de decir que el algoritmo heurístico que proponían (DC) era mucho más rápido que otro (H2) propuesto anteriormente por Miltenburg [3] y que daba los mismos resultados que éste. Yeomans, que ha publicado diversos trabajos con Miltenburg, parecía tomarse las afirmaciones de Ding y Cheng como una afrenta personal, y, aunque este espíritu de solidaridad con un colega sea loable, no por ello debe dejarse de lado la ecuanimidad propia del método científico. Yeomans decía que el trabajo de Ding y Cheng era irrelevante ya que el aumento de velocidad de su algoritmo no tenía importancia dado que H2 ya existía. A mayor abundamiento también existían otros procedimientos que proporcionaban una solución exacta. Un argumento parecido llevaría a rechazar el TAV dado que ya existen expresos y, si no son suficentes, también aviones.

Yeomans no decía nada de otros pecados de Ding y Cheng, más graves a nuestros ojos. Uno de ellos era el haber publicado al mismo tiempo el mismo artículo en dos revistas diferentes ([2] y [4]). El segundo pecado exige cierta explicación. Uno de nosotros había trabajado con H2 y DC y recordaba que las dos heurísticas no daban el mismo resultado en todos los casos. Recuperamos un caso en que ello ocurría, identificamos la diferencia, repasamos manualmente la aplicación de los algoritmos y llegamos a la conclusión de que dicha diferencia era intrínseca a los algoritmos y no debida a nuestra implementación, lo que ponía en cuarentena la argumentación de Yeomans. Analizando las causas detectamos un error en la demostración de Ding y Cheng, que hacía que DC y H2 no fuesen el mismo algoritmo en dos versiones si no dos algoritmos heurísticos diferentes. Además aunque H2 parecía acercarse en promedio más que DC a soluciones óptimas en ocasiones DC daba soluciones mejores.

Por tanto Yeomans había pontificado sobre DC partiendo de las aseveraciones de Ding y Cheng pero no realizando una experimentación adecuada, lo que a nuestros ojos era un fallo. Preparamos en consecuencia una Nota [5] comentando el artículo de Yeomans y la enviamos a la revista que había publicado uno de los artículos de Ding y Cheng y el comentario de Yeomans, "OR Letters".

Aunque esta revista blasona la rápidez de publicación, tardó 8 meses en enviarnos una carta en la que se declaraba dispuesta a considerar la publicación de nuestra nota si la acortábamos y teníamos en cuenta la opinión del "referee" (único) que había informado. Este "referee" aseguraba que nuestra nota sólo se refería a aspectos periféricos de los trabajos anteriores:

"... Thus the authors' comments only address peripheral

issues in both Ding & Cheng's paper and Yeomans' note."
Si periférico significa mostrar los errores existentes en ambas publicaciones estamos totalmente de acuerdo.

Las modificaciones dictadas por el "referee" significaban en definitiva que se toleraba que nos metieramos con Ding y Cheng pero no con Yeomans. Los padrinos de éste en "OR Letters" deben ser más poderosos que los de aquellos. El resultado fue otra Nota [6] que ha sido sometida y estamos en espera de los nuevos comentarios del "referee".

Una de las observaciones de éste merece un comentario final:

> "5. In your computational experiments how do you address Yeomans' criticism that empirical comparisons need to be made between large test problems (i.e., "n=20,D=5000") for such comparisons to be valid?"

que contrasta con otro "dictat":

> "2. Do not reiterate what Yeomans has already pointed out unless it is absolutely relevant to your paper....."

Encontrar la solución óptima para un problema de grandes dimensiones es costoso en tiempo, lo que no debe ignorar el "referee", por lo que nuestra respuesta, no incluída en el cuerpo del artículo ha sido:

> "We think, in agreement with Yeomans, that empirical comparisons need to be made between test problems as large as industrial problems are (for instance, "n = 16-20, D = 800-1000, although the size of industrial problems is actually very diverse), since the solution quality for certain heuristics deteriorate as the size of problem increases (Yeomans [6], p. 300). A sample of our results is:

		best heuristic value		total
problem 4_45				
672 instances	%	opt	non-opt	
	H2=DC	66.67	4.46	71.13
	H2<DC	21.88	1.19	23.07
	H2>DC	5.50	0.30	5.80
	total	94.05	5.95	100.00

		best heuristic value		total
problem 5_55				
3765 instances	%	opt	non-opt	
	H2=DC	53.86	13.25	67.11
	H2<DC	21.73	2.71	24.44
	H2>DC	6.19	2.26	8.45
	total	81.78	18.22	100.00

		best heuristic value		total
problem 6_80				
49342 instances	%	opt	non-opt	
	H2=DC	29.90	19.83	49.73
	H2<DC	21.94	11.38	33.32
	H2>DC	7.20	9.75	16.95
	total	59.04	40.96	100.00

We are developping further experiments with large test problems, but we think the validity of our comment about Ding & Cheng paper does not depend on the results of these experiments. We do not mention this issue in our note because it has already been pointed out by Yeomans in his paper."

Todos los indicios señalan que la mayor calidad de H2 respecto DC va reduciéndose a medida que aumenta la dimensión del problema. Es posible que para n=20 y D=5000 DC sea superior a H2 no sólo en rapidez sino en calidad de las soluciones. Dadas las circunstancias hemos callado esta sospecha.

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