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Modeling and Solving a Variant of *MMSP-W* Problem with Production Mix Restrictions

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Abstract In this paper, we propose a procedure based on Bounded Dynamic Programming (*BDP*) to solve the Mixed-Model Sequencing Problem with Work overload Minimisation (*MMSP-W*), with serial workstations, unrestricted (or free) interruption of the operations and with production mix restrictions.

Keywords: Sequencing, Work Overload, Production Mix, Linear Programming, Bounded Dynamic Programming

1.1 Introduction

In mixed-model manufacturing lines, which are common in Just-in-time (*JIT*) and Douki Seisan (*DS*) ideologies, several variants of one or more products can be handled. This flexibility determines the order in which the units are treated to drastically reduce intermediate stocks and to capitalise on the time available for manufacturing. In this context, there are three classes of sequencing mixed products problems (Boysen et al., 2009): (1) Mixed-model sequencing, (2) Car sequencing and (3) Level scheduling. The Mixed-Model Sequencing Problem with Work overload Minimisation (*MMSP-W*) (Yano and Rachamadugu, 1991; Scholl et al., 1998) belongs to the first class.

The *MMSP-W* consists of sequencing T products, of which d_i are of type i ($i = 1,\ldots,|I|$). A unit of product type i requires to each processor (operator, robot, etc..) of the workstation k ($k = 1,\ldots,|K|$) a standard processing time, $p_{i,k}$.

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The standard time assigned to each processor to work on any product unit is the cycle time c. When a cycle ends at the workstation k, it can work on the product in progress in an additional positive time $l_k - c$, being l_k the time window. When it is not possible to complete all of the work required by the demand plan, overload is generated. The objective of the problem is to maximise the total work performed which is equivalent to minimising the total overload generated (see Theorem 1 in Bautista and Cano, 2011).

On the other hand, the Level scheduling problems class focuses on obtaining regular sequences in production and consumption of components, among them are: (1) Product Rate Variation (*PRV*), which is used to preserve the production mix (Miltenburg, 1989) and (2) Output Rate Variation (*ORV*), based on the manner of sequencing the mixed products units, used at Toyota plants to maintain a constant consumption of components over time (Monden, 1983).

Our proposal is organised as follows. Section 1.2 extends the mathematical program proposed in Bautista et al. (2011) for the *MMSP-W*, with the incorporation of the preservation of the production mix. Section 1.3 presents a procedure based on Bounded Dynamic Programming (*BDP*) that combines features of dynamic programming with features of branch and bound algorithms (Bautista, 1993; Bautista et al., 1996). In section 1.4, we perform an experiment with reference instances using the *BDP* procedure and the Gurobi solver. Finally, some conclusions about this work, are collected in 1.5.

1.2 Model for MMSP-W with Serial Workstations and Unrestricted Interruption of the Operations and Production Mix Restrictions

For the *MMSP-W* with serial workstations, unrestricted interruption of the operations, production mix restrictions (*pmr*) and work overload minimisation, we take as reference the *M4'* model, proposed by Bautista et al. (2011). The parameters and variables of the extended model *M4'_pmr* are presented below.

Parameters

K	Set of workstations ($k = 1,...,\|K\|$)
b_k	Number of homogeneous processors at workstation k
I	Set of product types ($i = 1,...,\|I\|$)
d_i	Programmed demand of product type i
$p_{i,k}$	Processing time required by a unit of type i at workstation k for each homogeneous processor (at normal activity)
T	Total demand. Obviously: $\sum_{i=1}^{\|I\|} d_i = T$
t	Position index in the sequence ($t = 1,\ldots,T$)
c	Cycle time, the standard time assigned to workstations to process any product unit

l_k	Time window, the maximum time that the workstation k is allowed to work on any product unit, where $l_k - c > 0$ is the maximum time that the work in process is held at workstation k

Variables	
$x_{i,t}$	Binary variable equal to 1 if a product unit i ($i = 1,\ldots,\lvert I\rvert$) is assigned to the position t ($t = 1,\ldots,T$) of the sequence, and 0 otherwise
$s_{k,t}$	Start instant of the operation in t^{th} unit of the sequence of products at workstation k ($k = 1,\ldots,\lvert K\rvert$)
$w_{k,t}$	Overload generated for the t^{th} unit of the product sequence at workstation k for each homogeneous processor (at normal activity); measured in time.
$\hat{s}_{k,t}$	Positive difference between the start instant and the minimum start instant of the t^{th} operation at workstation k. $\hat{s}_{k,t} = [s_{k,t} - (t-1)c]^+$ (with $[x]^+ = \max\{0,x\}$)
$\rho_{k,t}$	Processing time required by the t^{th} unit of the sequence of products at workstation k

Model *M4'_pmr:*

$$Min \;\; W = \sum_{k=1}^{|K|}\left(b_k \sum_{t=1}^{T} w_{k,t}\right) \tag{1.1}$$

subject to:

$$\sum_{t=1}^{T} x_{i,t} = d_i \qquad \forall i = 1,\ldots,|I| \tag{1.2}$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \qquad \forall t = 1,\ldots,T \tag{1.3}$$

$$\rho_{k,t} = \sum_{i=1}^{|I|} p_{i,k}\, x_{i,t} \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.4}$$

$$\rho_{k,t} - w_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.5}$$

$$\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + \rho_{k,t-1} - w_{k,t-1} - c \qquad \forall k = 1,\ldots,|K|;\ \forall t = 2,\ldots,T \tag{1.6}$$

$$\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + \rho_{k-1,t} - w_{k-1,t} - c \qquad \forall k = 2,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.7}$$

$$\hat{s}_{k,t} + \rho_{k,t} - w_{k,t} \leq l_k \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.8}$$

$$\hat{s}_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.9}$$

$$w_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{1.10}$$

$$x_{i,t} \in \{0,1\} \qquad \forall i = 1,\ldots,|I|;\ \forall t = 1,\ldots,T \tag{1.11}$$

$$\hat{s}_{1,1} = 0 \tag{1.12}$$

$$\sum_{\tau=1}^{t} x_{i,\tau} \geq \left\lfloor t \cdot \frac{d_i}{T} \right\rfloor \qquad \forall i = 1,\ldots,|I|;\ \forall t = 1,\ldots,T \tag{1.13}$$

$$\sum_{\tau=1}^{t} x_{i,\tau} \leq \left\lceil t \cdot \frac{d_i}{T} \right\rceil \qquad \forall i = 1,\ldots,|I|;\ \forall t = 1,\ldots,T \tag{1.14}$$

Objective function (1.1) and constraints (1.2) to (1.12) correspond to the mathematical program *M4'* proposed in Bautista et al. (2011), while the constraints (1.13) and (1.14) are those that incorporate the preservation property of the production mix desired in *JIT* (Toyota) and *Douki Seisan* (Nissan) philosophies. Therefore, an alternative to formulate our problem is to replace the objective function (1.1) by the following bi-objective function:

$$Min \;\; W = \sum_{k=1}^{|K|}\left(b_k \sum_{t=1}^{T} w_{k,t}\right) \quad \wedge \quad Min \;\; \Delta_Q(X)) = \sum_{i=1}^{|I|}\sum_{t=1}^{T}\left(\sum_{\tau=1}^{t} x_{i,\tau} - t\frac{d_i}{T}\right)^2 \tag{1.15}$$

1.3 *BDP* for the *MMSP-W*

This section presents the basic elements of the *BDP* procedure applied to the resolution of *MMSP-W* with serial workstations, unrestricted interruption of the operations and production mix restrictions.

1.3.1 Global and Partial Bounds

Similar to Bautista et al. (2011), to obtain the bounds of the overloads associated to partial sequence $\pi(t)=\{\pi_1,\pi_2,...,\pi_t\}$ and a partial bound for the complement $R(\pi(t))$ associated to the sequence or segment $\pi(t)$, we impose the following conditions to *M4'_pmr*: (1) the values of the variables $x_{i,\tau}$ ($i=1,\ldots,|I|$; $\tau=1,\ldots,t$) are fixed by $\pi(t)$ and (2) the binary condition is relaxed for the variables $x_{i,\tau}$ ($i=1,\ldots,|I|$; $\tau=t+1,\ldots,T$).

The result is the following linear program, *LB_M4'_pmr*:

$$Min \quad LB(W(\pi(t)))=\sum_{k=1}^{|K|}\left(b_k\sum_{t=1}^{T}w_{k,t}\right) \tag{1.16}$$

Subject to:

(1.2) – (1.10) and (1.12) – (1.14) from *M4'_pmr*

$$x_{\pi_\tau,\tau}=1 \qquad \forall\tau=1,\ldots,t \tag{1.17}$$

$$0\le x_{i,\tau}\le 1 \qquad \forall i=1,\ldots,|I|;\ \forall\tau=t+1,\ldots,T \tag{1.18}$$

The previous linear program provides an overall bound of the total overload $(LB(W(\pi(t))))$, the value of the overload associated to the segment $\pi(t)$ $(W(\pi(t)))$ and a bound of the overload associated to the complement $R(\pi(t))$ $(LB(R(\pi(t))))$. These values can be calculated as follows:

$$W(\pi(t))=\sum_{k=1}^{|K|}\left(b_k\sum_{\tau=1}^{t}w_{k,\tau}\right) \tag{1.19}$$

$$LB(R(\pi(t)))=\sum_{k=1}^{|K|}\left(b_k\sum_{\tau=t+1}^{T}w_{k,\tau}\right) \tag{1.20}$$

The relative completion instants ($\hat{e}_{k,t}$) of the last operation of the partial sequence $\pi(t)$, in each workstation, can be obtained as follows:

$$\hat{e}_{k,t}=\hat{s}_{k,t}+\rho_{k,t}-w_{k,t} \qquad \forall k=1,\ldots,|K| \tag{1.21}$$

1.3.2 Graph Associated with the Problem

Similar to Bautista and Cano (2011) and Bautista et al. (2011), we can build a linked graph without loops or direct cycles of $T+1$ stages. The set of vertices in level $t\ (t=0,\ldots,T)$ will be noted as $J(t)$. $J(t,j)$ $(j=1,\ldots,|J(t)|)$ being a vertex of level t, which will be represented as follows:

$$J(t,j)=\left\{(t,j),\ \vec{q}(t,j),\ \pi(t,j),\ LB\left(W\left(\pi(t,j)\right)\right),\ \vec{e}(t,j),\ \vec{e}^{\,c}(t,j)\right\} \tag{1.22}$$

where:

- $\vec{q}(t,j)=(q_1(t,j),\ldots,q_{|I|}(t,j))$ represents the vector of demand satisfied.
- $\vec{e}(t,j)=(e_1(t,j),\ldots,e_{|K|}(t,j))$ is the vector of absolute completion instants of the operations at the workstations and $\vec{e}^{\,c}(t,j)=(e_1^c(t,j),\ldots,e_{|K|}^c(t,j))$ is the vector of corrected completion instants in accordance with the cycle time.
- $\pi(t,j)$ represents the sequence of t units of product associated to the vertex.
- $LB(W(\pi(t,j)))$ is the bound of the overall overload of the sequence $\pi(t,j)$, given by the linear programs *LB_M4'* (Bautista et al., 2011) and *LB_M4'_pmr*.

And the vertex $J(t,j)$ has the following properties:

$$\sum_{i=1}^{|I|} q_i(t,j)=t \tag{1.23}$$

$$\left\lfloor \frac{d_i}{T}\cdot t\right\rfloor \le q_i(t,j)\le \left\lceil \frac{d_i}{T}\cdot t\right\rceil \tag{1.24}$$

$$e_k^c(t,j)=\max\left\{(t+k-2)c+\hat{e}_{k,t}\,,(t+k-1)c\right\} \tag{1.25}$$

At level 0 of the graph, there is only one $J(0)$ vertex. Initially, we may consider that at level t, $J(t)$ contains the vertices associated to all of the sub-sequences that can be built with t products that satisfy properties (1.23) – (1.25). However, it is easy to reduce the cardinal that $J(t)$ may present a priori, establishing the following definition of pseudo-dominance:

$$\pi(t,j_1)\prec\pi(t,j_2)\Leftrightarrow\left\{\begin{array}{c}\left[\vec{q}(t,j_1)=\vec{q}(t,j_2)\right]\wedge\left[W(\pi(t,j_1))\le W(\pi(t,j_2))\right]\wedge\\ \left[\vec{e}^{\,c}(t,j_1)\le\vec{e}^{\,c}(t,j_2)\right]\end{array}\right\} \tag{1.26}$$

1.3.3 The Use of BDP

For this study, we used a procedure based on *BDP*. This procedure combines features of dynamic programming with features of branch and bound algorithms. The principles of *BDP* have been described by Bautista (1993) and Bautista et al. (1996).

The procedure has the following functions: (1) Select_vertex (t): selects, following a nondecreasing ordering of the $LB(W(\pi(t-1,j)))$ values, one of the vertices consolidated in stage $t-1$; (2) Develop_vertex (t): develops the selected vertex in previous function adding a new product unit with pending demand; (3) Filter_vertices ($Z_0, H, LBZ_{\min}$): chooses, from all the vertices developed in the previous function, a maximum number H of the most promising vertices (according to the lowest values of $LB(W(\pi(t,j)))$), and removing those vertices in which their lower bound is greater than Z_0 (known initial solution); and (4)

End_stage (): consolidates the most promising vertices in stage t (H vertices as maximum).

The scheme of the procedure is described below (Bautista and Cano, 2011):

BDP – MMSPW
Input: $T, |I|, |K|, d_i(\forall i), l_k, b_k(\forall k), p_{ik}(\forall i, \forall k), c, Z_0, H$
Output: list of sequences obtained by *BDP*

```
Initialization: t = 0; LBZ_min = ∞
While (t < T) do
    t = t+1
    While (list of consolidated vertices in stage t-1 not empty) do
        Select_vertex (t)
        Develop_vertex (t)
        Filter_vertices ( Z_0, H, LBZ_min )
    end while
    End_stage ()
end while
end BDP – MMSPW
```

1.4 Computational Experiment

225 reference instances (Bautista and Cano, 2008) are used (see tables 2 and 3 from Bautista and Cano, 2011), which are built from 45 demand plans grouped in 5 blocks (*B*) and 5 process time structures (*E*).

To obtain the optimal solutions for the instances from models *M4'* and *M4'_pmr,* the Gurobi v4.5.0 solver was used. Those solutions were compared with the solutions offered by the proposed *BDP* procedures from both models, *M4'* and *M4'_pmr*, under the following conditions: (1) *BDP* procedure programmed in C++, using gcc v4.2.1, (2) five windows width (*H*) were used, with values 1, 6, 16, 32, 64. The initial solution, Z_0, for each window width was the solution obtained by *BDP* with the previous window width, except for $H = 1$, where Z_0 was established as ∞; and (3) to calculate the lower bounds, $LB(W(\pi(t,j)))$, of the overload associated to each vertex in the *BDP* procedure, the Gurobi v4.5.0 solver was used, solving the linear programs associated to *LB_M4'* and *LB_M4'_pmr*. Both procedures have been run on an Apple Macintosh iMac computer with an Intel Core i7 2.93 GHz processor and 8 GB RAM using MAC OS X 10.6.7.

The results obtained by the experiment are collect in tables 1.1 and 1.2.

Table 1.1 shows that the incorporation of restrictions (1.13) and (1.14), into the original model *M4*' (Bautista et al., 2011), to preserve the production mix, reduces the average CPU time required to obtain the optimal solutions using Gurobi by a factor of five; also we can see that the CPU time needed by *BDP* to obtain the best solutions with a window width of 64, is reduced by half. Additionally, regarding the average CPU time, with *M4'* model the *BDP* is 40 times faster than Gurobi, and 15 times faster with *M4'_pmr* model.

Table 1.1 Minimum, maximum and average CPU times needed to obtain optimal solutions for the 225 instances given by models *M4'* and *M4'_pmr* using Gurobi and *BDP*.

	$M4'_{Gurobi}$	$M4'\_pmr_{Gurobi}$	$M4'_{BDP}$	$M4'\_pmr_{BDP}$
$CPU_{\min}$	0.04	0.03	0.04	0.06
$CPU_{\max}$	2224.98	110.53	5.50	2.72
$\overline{CPU}$	59.95	11.79	1.58	0.78

Table 1.2 RPD_1, RPD_2 and RPD_3 values by structures, blocks and average (225 instances), of the solutions, given by *M4'* and *M4'_pmr*, of W and $\Delta_Q(X)$ from Gurobi and *BDP*.

	W			$\Delta_Q(X)$		
	RPD_1	RPD_2	RPD_3	RPD_1	RPD_2	RPD_3
E1	-3.74	-3.11	-0.09	46.89	39.62	-6.22
E2	-1.25	-1.06	-0.02	25.79	25.15	0.82
E3	-0.69	0.24	0.00	34.24	46.91	0.56
E4	-0.01	0.36	0.00	36.16	63.17	-1.25
E5	-1.00	-0.69	-0.01	23.10	32.92	-0.90
B1	-0.04	-0.04	0.00	17.45	32.52	-3.87
B2	-1.31	-1.14	0.00	25.33	35.62	-2.80
B3	-2.57	-1.31	0.00	45.49	46.19	-0.56
B4	-1.84	-1.57	-0.11	41.44	40.21	-2.00
B5	-1.12	-0.66	-0.03	32.90	43.41	-0.78
Average	-1.34	-0.85	-0.03	33.24	41.55	-1.40

Regarding the quality of the solutions, in each instance, we take as starting point the best solution (minimum) given by *M4'* and *M4'_pmr* using Gurobi and *BDP*. From these solutions, we determine three types of relative percentage deviations (*RPD*) applied to W and $\Delta_Q(X)$: RPD_1 compares the solutions offered by *M4'* and *M4'_pmr* with Gurobi, RPD_2 compares the solutions offered by *M4'* and *M4'_pmr* with *BDP* and RPD_3 compares the solutions offered by *M4'_pmr* with both procedures.

Table 1.2 shows: (1) using Gurobi, the solutions for overall overload (*W*) offered by *M4'_pmr* are worse, by an average of 1.34%, than those offered by *M4'*, and a 0,85% using *BDP*; (2) due to the pseudo-dominances (1.26) the overall overload offered by *M4'_pmr* through the *BDP* is, on average 0.03% worse, compared to the solutions obtained by Gurobi; (3) the incorporation of constraints (1.13) and (1.14) into *M4'*, gives improvements in the preservation of production mix by an average of 33.24% and 41.55% using Gurobi and *BDP*, respectively; and (4) the performance of Gurobi is insignificantly better than *BDP* with respect to the preservation of mix production by an average of 1.40%.

1.5 Conclusions

We presented the model *M4'_pmr* that corresponds to the *MMSP-W* problem with serial workstations, unrestricted interruption of the operations, with production mix restrictions (*pmr*) and work overload minimisation, taking as reference the model *M4'*, proposed by Bautista et al. (2011).

For the new problem, we propose two methods of resolution: mathematical programming (Gurobi v4.5.0 solver) and bounded dynamic programming (*BDP*).

A computational experience is made with 225 intances from the literature. All the optima are obtained with Gurobi for both models. In addition, these instances are solved with *BDP* (*H*=64) reaching 175 optimal solutions through *M4'* (average worsening of 0.51%) and 221 optima through *M4'_pmr*, due to the pseudo-dominances.

In average CPU times, *BDP* spends, on average, in *M4'*, a fortieth of the time spent by Gurobi, and a fifteenth in *M4'_pmr*. In addition, the incorporation, into *M4'*, of the production mix restrictions, reduces to one fifth of the average CPU time with Gurobi and in half with *BDP*.

The worsening, in overall overload, by an average of 1.34% and 0.85% of *M4'_pmr* over *M4',* obtained by Gurobi and *BDP*, respectively, are offset by the gains of preservation of production mix of 33.24% (Gurobi) and 41.55% (*BDP*).

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